Quantum Zeno effect and the detection of gravitomagnetism.

A. Camacho * †
Astrophysikalisches Institut Potsdam.
An der Sternwarte 16, D–14482 Potsdam, Germany.

Abstract

In this work we introduce two experimental proposals that could shed some light upon the inertial properties of intrinsic spin. In particular we will analyze the role that the gravitomagnetic field of the Earth could have on a quantum system with spin 1/2. We will deduce the expression for Rabi transitions, which depend, explicitly, on the coupling between the spin of the quantum system and the gravitomagnetic field of the Earth. Afterwards, the continuous measurement of the energy of the spin 1/2 system is considered, and an expression for the emerging quantum Zeno effect is obtained. Thus, it will be proved that gravitomagnetism, in connection with spin 1/2 systems, could induce not only Rabi transitions but also a quantum Zeno effect.

*email: acamacho@aip.de

†This essay received an "honorable mention" in the Annual Essay Competition of the Gravity Research Foundation for the year 2000 — Ed.

1 Introduction.

In more than three–quarters of a century the theory of general relativity (GR) has achieved a great experimental triumph. Neverwithstanding, at this point it is also important to comment that all the current direct confirmations of GR are confirmations of weak field corrections to the Galilei–Newton mechanics [1]. We must also add that one of the most important, and yet undetected, predictions of GR is the so called gravitomagnetic field [1], sometimes also called Lense–Thirring effect [2], which is generated by mass–energy currents. Its measurement would constitute a direct experimental evidence against an absolute inertial frame of reference, and would at the same time show the basic role that local inertial frames play in nature, i.e., it would be a direct proof that local inertial frames are influenced and dragged by mass–energy currents relative to other mass.

The first efforts in the detection of this gravitomagnetic field are quite old [3] and have already included many interesting proposals [4, 5, 6].

An additional topic in connection with gravitomagnetism is related to its coupling with intrinsic spin, this issue is of fundamental interest since it comprises the inertial properties of intrinsic spin. It is noteworthy to comment that this point is under constant analysis [7].

In this work we introduce two experimental proposals that could lead to the detection of the coupling between intrinsic spin and the gravitomagnetic field. We analyze the role that the gravitomagnetic field of the Earth could have on a quantum system with spin 1/2, i.e., our results could allow us to confront the effects of mass–energy density currents upon spin. In particular we deduce a Rabi formula, which depends on the coupling between the spin of the quantum system and the gravitomagnetic field of the Earth. Afterwards, the continuous measurement of the energy of the spin 1/2 system is considered, and a Zeno effect is obtained.

2 Rabi transitions and the gravitomagnetic field.

Let us consider a spin 1/2 system immersed in the gravitational field of a rotating uncharged, idealized spherical body with mass M and angular momentum J. In the weak field and slow motion limit the metric, in the Boyer–Lindquist coordinates, reads [8]

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2r}\right)dt^2 + \left(1 - \frac{2GM}{c^2r}\right)^{-1}dr^2$$

$$+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)-\frac{4GJ}{c^2r}\sin^2\theta d\phi dt. \tag{1}$$

The gravitomagnetic field in this case is approximately [1]

$$\vec{B}=2\frac{G}{c^2}\frac{\vec{J}-3(\vec{J}\cdot\hat{x})\hat{x}}{|\vec{x}|^3}. \tag{2}$$

We will assume that the expression that describes the precession of orbital angular momentum, immersed, for instance, in the gravitational field of the Earth, can be also used for the description of the dynamics in the case of intrinsic spin. This is a natural extension of general relativity [7].

Let us now denote the angular momentum of our spherical body by $\vec{J}=J\hat{z}$, being $\hat{z}$ the unit vector along the direction of the angular momentum. Our quantum particle is prepared such that $\vec{S}=S_z\hat{z}$, it has vanishing small velocity and acceleration, and it is located on the z–axis, with coordinate Z.

There is a formal analogy between the weak field and slow motion of the gravitomagnetic field in general relativity and the magnetic field in electromagnetism [1]. Following this analogy we may write down the interaction Hamiltonian (acting in the two–dimensional spin space of our spin 1/2 system), which gives the coupling between $\vec{B}$ and the spin, $\vec{S}$, of our particle

$$H=-\vec{S}\cdot\vec{B}. \tag{3}$$

Introducing expression (2) we may rewrite the interaction Hamiltonian as follows

$$H=2\frac{GJ\hbar}{c^2Z^3}\left[|+><+|-|-><-|\right]. \tag{4}$$

Here $|+>$ and $|->$ represent the eigenkets of S_z. Clearly, the introduction of the gravitomagnetic field renders two energy states

$$E_{(+)} = 2\frac{GJ\hbar}{c^2 Z^3}, \tag{5}$$

$$E_{(-)} = -2\frac{GJ\hbar}{c^2 Z^3}, \tag{6}$$

where $E_{(+)}$ ($E_{(-)}$) is the energy of the spin state $+\hbar/2$ ($-\hbar/2$). Let us now define the frequency

$$\Omega = \left(E_{(+)} - E_{(-)}\right)/\hbar = 4\frac{GJ}{c^2 Z^3}. \tag{7}$$

The present analogy allows us to consider the emergence of Rabi transitions [9]. In order to do this let us now introduce a rotating magnetic field, which, at the point where the particle is located, has the following form

$$\vec{b} = b\left[\cos(wt)\hat{x} + \sin(wt)\hat{y}\right], \tag{8}$$

where $\hat{x}$ and $\hat{y}$ are two unit vectors perpendicular to the z–axis, and b is a constant magnetic field.

Under these conditions the total Hamiltonian reads

$$\begin{aligned} H_T = 2\frac{GJ\hbar}{c^2 Z^3}\left[|+><+| - |-><-|\right] \\ -\frac{eb\hbar}{2mc}\left[e^{-iwt}|+><-| + e^{iwt}|-><+|\right]. \end{aligned} \tag{9}$$

Looking for a solution in the form $|\alpha >= c_{(+)}(t)|+> + c_{(-)}(t)|->$, we find the usual situation [9] (our quantum system has been initially prepared such that $c_{(-)}(0) = 1$ and $c_{(+)}(0) = 0$.)

$$c_{(-)}(t) = \exp\left[-i\frac{E_{(-)}}{\hbar}t \ + \frac{i}{2}(w-\Omega)t\right]\left[\cos(\Gamma t) - i\frac{(w-\Omega)}{2\Gamma}\sin(\Gamma t)\right], \qquad (10)$$

$$c_{(+)}(t) = i\frac{eb}{2mc\Gamma}\exp\left[-i\frac{E_{(+)}}{\hbar}t \ - \frac{i}{2}(w-\Omega)t\right]\sin(\Gamma t). \qquad (11)$$

where $\Gamma = \sqrt{(\frac{eb}{2mc})^2 + \frac{(w-\Omega)^2}{4}}$.

In this way we find

$$\frac{|c_{(-)}(t)|^2}{|c_{(-)}(t)|^2 + |c_{(+)}(t)|^2} = \left[1 + \frac{(\frac{eb}{2mc\Gamma})^2\sin^2(\Gamma t)}{\cos^2(\Gamma t) + \frac{(w-\Omega)^2}{4\Gamma^2}\sin^2(\Gamma t)}\right]^{-1}. \qquad (12)$$

Clearly, the Rabi transitions depend upon the coupling between spin and the gravitomagnetic field.

$$\left(4\frac{GJ}{c^2Z^3} - w\right)^2 = 4\left[\Gamma^2 - \left(\frac{eb}{2mc}\right)^2\right]. \qquad (13)$$

3 Quantum Zeno effect and gravitomagnetism.

Let us now measure, continuously, the energy of our spin 1/2 system, such that E is the measurement output, and that this experiment lasts a time T. This kind of measuring process can be described by the so called effective Hamiltonian formalism [10, 11], which is one of the models that exist in the topic of quantum measurement theory [12]. In our case the corresponding effective Hamiltonian reads

$$
\begin{aligned}
H_{eff} = 2\frac{GJ\hbar}{c^2Z^3}\left[1 + i\frac{2\hbar}{T\Delta E^2}\left(E - \frac{GJ\hbar}{c^2Z^3}\right)\right]|+><+| \\
-2\frac{GJ\hbar}{c^2Z^3}\left[1 + i\frac{2\hbar}{T\Delta E^2}\left(E + \frac{GJ\hbar}{c^2Z^3}\right)\right]|-><-| \\
-\frac{eb\hbar}{2mc}\left[e^{-iwt}|+><-| + e^{iwt}|-><+|\right] - i\frac{E^2\hbar}{T\Delta E^2}\Pi,
\end{aligned}
\tag{14}
$$

where Π is the unit operator in the spin space of our particle. Looking for solutions with the form $|\alpha>= c_{(+)}(t)|+> + c_{(-)}(t)|->$, we deduce

$$
\begin{aligned}
c_{(-)}(t) = \exp\left[-i\frac{E_{(-)}}{\hbar}t - \frac{(E_{(-)} - E)^2}{T\Delta E^2}t + i\tilde{\Gamma}t\right] \\
\times\left[c_{(-)}(0)\cos(\beta t) - i\frac{c_{(-)}(0)\tilde{\Gamma} + (\gamma/\hbar)c_{(+)}(0)}{\beta}\sin(\beta t)\right],
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
c_{(+)}(t) = \exp\left[-i\frac{E_{(+)}}{\hbar}t - \frac{(E_{(+)} - E)^2}{T\Delta E^2}t - i\tilde{\Gamma}t\right] \\
\times\left[c_{(+)}(0)\cos(\beta t) + i\frac{c_{(+)}(0)\tilde{\Gamma} - (\gamma/\hbar)c_{(-)}(0)}{\beta}\sin(\beta t)\right],
\end{aligned}
\tag{16}
$$

where $\tilde{\Gamma} = \frac{(w-\Omega)}{2} + \frac{i}{2T\Delta E^2}\left[(E_{(+)} - E)^2 - (E_{(-)} - E)^2\right]$, $\beta^2 = (\gamma/\hbar)^2 + \tilde{\Gamma}^2$, and finally $\gamma = -\frac{eb\hbar}{2mc}$.

Let us now suppose that the measurement output is the energy of the ground state, $E_{(-)}$, that we have a resonant perturbation, and that initially only the lowest energy state was populated, in other words, $E = E_{(-)}$, $\hbar w = E_{(+)} - E_{(-)}$, and $c_{(-)}(0) = 1$, $c_{(+)}(0) = 0$.

Hence (15) and (16) become

$$
c_{(-)}(t) = \exp\left[-i\frac{E_{(-)}}{\hbar}t - \frac{(E_{(+)} - E_{(-)})^2}{2T\Delta E^2}t\right]\left[\cos(\beta t) - i\frac{\tilde{\Gamma}}{\beta}\sin(\beta t)\right],
\tag{17}
$$

$$c_{(+)}(t) = -i\frac{\gamma}{\beta\hbar}\exp\left[-i\frac{E_{(+)}}{\hbar}t\ -\frac{(E_{(+)}-E_{(-)})^2}{2T\Delta E^2}t\right]\sin(\beta t). \qquad (18)$$

Let us now assume that $\frac{(E_{(+)}-E_{(-)})^4}{4T^2\Delta E^4} > \gamma^2/\hbar^2$, then

$$P_{(-)}(t) = \left[1 + \frac{\sinh^2(\frac{\gamma}{\hbar}\tilde{\Omega}t)}{\tilde{\Omega}^2[\cosh(\frac{\gamma}{\hbar}\tilde{\Omega}t) + \frac{\hbar(E_{(+)}-E_{(-)})^2}{2T\gamma\tilde{\Omega}\Delta E^2}\sinh(\frac{\gamma}{\hbar}\tilde{\Omega}t)]^2}\right]^{-1}, \qquad (19)$$

where $\tilde{\Omega} = \sqrt{\frac{\hbar^2(E_{(+)}-E_{(-)})^4}{4T^2\gamma^2\Delta E^4} - 1}$, $\gamma = -\frac{eb\hbar}{2mc}$, and $P_{(-)}(t) = \frac{\vert c_{(-)}(t)\vert^2}{\vert c_{(-)}(t)\vert^2+\vert c_{(+)}(t)\vert^2}$.

In the case $t \to \infty$ this last expression reduces to

$$P^{(\infty)}_{(-)} = \left[1 + \left(\frac{c^2Z^3}{4GJ\hbar}\right)^2\frac{ebT\Delta E^2}{mc}\left(\sqrt{1-(\frac{c^2Z^3}{4GJ\hbar})^4(\frac{ebT\Delta E^2}{mc})^2} - 1\right)^{-2}\right]^{-1}. \qquad (20)$$

Clearly, Rabi transitions are inhibited, and the asymptotic value that here appears depends explicitly upon the coupling between intrinsic spin and the gravitomagnetic field, i.e., J emerges in expression (20).

At this point it must be commented that the behavior of spin leads, in some cases, to the emergence of a non–geometric element in gravity [13].

In this work Ahluwalia has considered two different classes of flavor–oscillation clocks. The first one comprises the superposition of different mass eigenstates, associated to a quantum test particle, such that all the terms of the corresponding superposition have the same spin component. The second class of flavor–oscillation clocks, contains, at least, two distinct spin projections.

If the gravitomagnetic field is absent, then both clocks redshift identically in the corresponding gravitational field. Nevertheless, if the source of the gravitational field has a nonvanishing angular momentum, then these redshifts do not coincide any more . This fact depends not only upon the gravitomagnetic component of the gravitational

field, but also on the quantum mechanical features of the employed quantum test particle. In other words, here a non–geometric element appears when gravitational and quantum mechanical phenomena are considered simultaneously.

Clearly, in the present essay we have a quantum system with spin immersed in a nonvanishing gravitomagnetic field. Nevertheless, our case is an eigenstate of the spin operator S_z, something that in Ahluwalia's second class of flavor–oscillation clocks does not happen. This last remark means that our quantum system is closer to Ahluwalia's first class of flavor–oscillation clocks than to his second one.

Finally, we must add that it is now possible to test, experimentally, the quantum Zeno effect [12], particularly using Penning traps to analyze Rabi transitions [14].

Acknowledgments.

The author would like to thank A. A. Cuevas–Sosa his help, and D.-E. Liebscher for the fruitful discussions on the subject. It is also a pleasure to thank R. Onofrio for bringing Refs. 6 and 11 to my attention. The hospitality of the Astrophysikalisches Institut Potsdam is also kindly acknowledged. This work was supported by CONACYT Posdoctoral Grant No. 983023.

References

[1] I. Ciufolini and J. A. Wheeler, "Gravitation and Inertia," Princeton University Press, Princeton, New Jersey, (1995).

[2] J. Lense and H. Thirring, Phys. Z. **19**, 711–750 (1918).

[3] B. and I. Friedländer, "Absolute and relative Bewegung," Simion–Verlag, Berlin, (1896).

[4] V. B. Braginsky, A. G. Polnarev, and K. S. Thorne, Phys. Rev Lett. **53**, 863–866 (1984).

[5] S. Vitale, M. Bonaldi, P. Falferi, G. A. Prodi, and M. Cerdonio, Phys. Rev. **B39**, 11993–12002 (1989).

[6] I. Ciufolini, E. Pavlis, F. Chieppa, E. Fernandes–Vieira, and J. Pérez–Mercader, Science **279**, 2100–2103 (2000).

[7] B. Mashhoon, Gen. Rel. Grav. **31**, 681–691 (1999).

[8] R. H. Boyer and R. W. Lindquist, J. Math. Phys. **8**, 265–281 (1967).

[9] J. J. Sakurai, "Modern Quantum Mechanics," Addison-Wesley Publishing Company, Reading, Mass. (1995).

[10] M. B. Mensky, "Continuous Quantum Measurements and Path Integrals," IOP, Bristol and Philadelphia (1993).

[11] R. Onofrio, C. Presilla, and U. Tambini, Phys. Lett. **A183**, 135–140 (1993).

[12] C. Presilla, R. Onofrio, and U. Tambini, Ann. Phys. **248**, 95–121 (1996).

[13] D. V. Ahluwalia, Gen. Rel. Grav. **29**, 1491-1501 (1997).

[14] W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. **D41**, 2295–2302 (1990).